Effects of a type of quenched randomness on car accidents in a cellular automaton model

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In this paper we numerically study the probability P_{ac} of the occurrence of car accidents in the Nagel-Schreckenberg (NS) model with a defect. In the deterministic NS model, numerical results show that there exists a critical value of car density below which no car accident happens. The critical density ρ_{c1} is not related only to the maximum speed of cars, but also to the braking probability at the defect. The braking probability at a defect can enhance, not suppress, the occurrence of car accidents when its value is small. Only the braking probability at the defect is very large, car accidents can be reduced by the bottleneck. In the nondeterministic NS model, the probability P_{ac} exhibits the same behaviors with that in the deterministic model except the case of $v_{\max}=1$ under which the probability P_{ac} is only reduced by the defect. The defect also induces the inhomogeneous distribution of car accidents over the whole road. Theoretical analyses give an agreement with numerical results in the deterministic NS model and in the nondeterministic NS model with $v_{max}=1$ in the case of large defect braking probability.

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I. INTRODUCTION

Recently, traffic problems have attracted much attention of a community of physicists. A number of models have been proposed to investigate the dynamical behavior of the traffic flow, including car-following models, cellular automata (CA) models, gas-kinetic models, and hydrodynamic models [1–3]. These dynamical approaches revealed complex physical phenomena of traffic flow among which are hysteresis, synchronization of flow, wide moving jams, and phase transitions, etc. Among these approaches, CA models can be used very efficiently for computers to perform real simulations [1]. Presently, based on the two basic CA models describing one-lane traffic flow, the Nagel-Schreckenberg (NS) model [4] and the Fukui-Ishibasi (FI) model [5], many CA models have been extended to investigate real traffic systems such as road blocks and hindrances, two-level crossing, highway junctions, etc. [1,6].

On the other hand, traffic jams and traffic accidents have become significant problems in a modern society. In recent years, the CA models have been extended to investigate the occurrence of traffic accidents [7–14]. With the help of the conditions for the occurrence of car accidents proposed by Boccara *et al.* [7], simulations of the probability for car accidents to occur have been offered in the NS models with periodic boundary or open boundary and in the FI model [8–11]. And analytical expressions for car accidents have also been provided in special cases [12]. Moreover, the probability for the occurrence of accidents has been studied in the velocity effect CA model and the two-lane CA model [13,14], respectively.

In this paper, we study the probability for car accidents to happen in the NS model with a bottleneck. CA models with a bottleneck have been extensively investigated [15–21], because bottlenecks often happen in real traffic systems, but the accident probability has not been explored so far. According to the previous studies of car accidents [7,10], the probability of car accidents is related to stopped cars and traffic flow, the studies on the probability of the occurrence of car accidents can lead us to understand traffic flow. On the other hand, because the bottleneck results in a distinct variety of traffic flow such as a saturated flow observed in a plateau region in the fundamental diagram, and vehicle queuing, changes of traffic flow, and number of stopped cars will certainly influence the probability of the occurrence of car accidents. Therefore car accidents in a CA model with a bottleneck should be further investigated.

The paper is organized as follows. Section II is devoted to the description of the model and the conditions for the occurrence of accidents. In Sec. III, the numerical studies of the car accidents are given, and effects of the stochastic braking and speed limiting on the probability of car accidents are considered. An accident probability at sites and the length of the sites where car accidents occur are also presented, respectively. Finally, the results are summarized in Sec. IV.

II. MODEL AND CAR ACCIDENTS

Our studies are based on the NS model which is defined on a single-lane road of L cells of equal size numbered by $i=1,2,\ldots,L$ and the time is discrete. Each site can be either empty or occupied by a car with the integer speed $v=0,1,2,\ldots,v_{\max}$, where $v_{\max}$ is the speed limit. Let d denote the number of empty cells in front of a car. The following four steps for all cars are performed in parallel with periodic boundary:

(1) Acceleration: $v\to\min(v+1,v_{\max})$.

(2) Slowing down: $v\to\min(d,v)$.

(3) Stochastic braking: if $v>0$, $v\to v-1$ with the probability p.

(4) Movement: move car v sites forward.

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A selected cell is designated as the bottleneck, on which each vehicle reduces the velocity by 1 with a probability p_d. Apparently, if $p_d = p$, the model returns to the NS model. In general, the value of the probability p_d at the defect is larger than the value of the probability p in the bulk.

In the basic NS model, car accidents will not occur, because the second rule of the update is designed to avoid accidents. The safety distance of the drivers is respected in the driving scheme. However, if the safety distance is not kept, due to the careless driving of the drivers, car accidents happen most likely. Based on the assumption, Boccara *et al.* [7] proposed that when three conditions (i) $d \leqslant v_{max}$, (ii) $v(i+1,t) > 0$, and (iii) $v(i+1,t+1) = 0$ are satisfied simultaneously, then car i will collide with the front car with a probability p' and a car accident happens.

Considering the effect of the stochastic braking on car accidents, one of our authors has proposed the modified accident conditions which are applicable for both deterministic and nondeterministic systems to correctly determine car accidents caused by careless drivers [10]. The first condition is that over the iterations of the rules (1)–(3), the velocity of the car is exactly equal to the number of empty cells in front of it, which means that the car can reach the position of the car ahead if the velocity of the car driven by the careless driver increases by one unit. The second condition is the moving car ahead. The third condition is that the moving car ahead is suddenly stopped. If the above three conditions are satisfied simultaneously and the velocity of following cars increases by 1 with the probability p', the collision between the two cars will occur.

Later, Moussa has investigated the effect of the delayed reaction time on the probability of car accidents in the NS model [22]. And Jiang *et al.* have modified the conditions for car accidents [23]. Detail analyses exhibit that the conditions of Jiang *et al.* for car accidents are similar with ours except in the case of $v_{\max} = 1$. In the case of $v_{\max} = 1$, they claimed that no car accident happens because the speed of cars can never exceed its limit. Obviously, this result does not accord with the simulated reality. In fact, traffic accidents will happen only if the conditions for accidents to occur are met, and will be independent of the velocity limit of cars.

In this paper, we only consider traffic accidents induced by inattention of drivers. If a driver accelerates stochastically and ignores the safety distance, a traffic accident is likely to occur. On the other hand, if reaction time of drivers is delayed, even the safety distance of the drivers is respected, a car accident will also take place. In the NS model, the time step of the update is fixed. Therefore delayed reaction time of careless drivers means that there is the additional velocity of the careless drivers after the iteration of the time step and the velocity of careless drivers increases accordingly, compared with the speed of other drivers. Thus $v(i,t+1) = v_{\max} + 1$ does not imply that the velocity of cars can exceed its limit, and only indicates the fact that the drivers are careless.

Thus in this paper we utilize our conditions to simulate traffic accidents. In the process of simulations, car accidents do not really happen. When the three necessary conditions are met simultaneously, the dangerous situations of the occurrence of car accidents exist. These dangerous situations are calculated and considered as the signal of the occurrence of accidents. Usually, the probability per car per time step for car accidents to occur is denoted by P_{ac}. Because the occurrence of car accidents is proportional to the occurrence of dangerous situations, and the proportional constant is p', the probability of traffic accidents can be scaled by the parameter p'. Here, the system size $L = 1000$ is selected and the results are obtained by averaging over 50 initial configurations and 2×10^3 time steps after discarding 8×10^3 initial transient states. A site of defect is selected at $L/2$. The model contains four basic parameters: the speed limit, the stochastic braking probability p, the average density $\rho = N/L$, and the braking probability at the defect p_d.

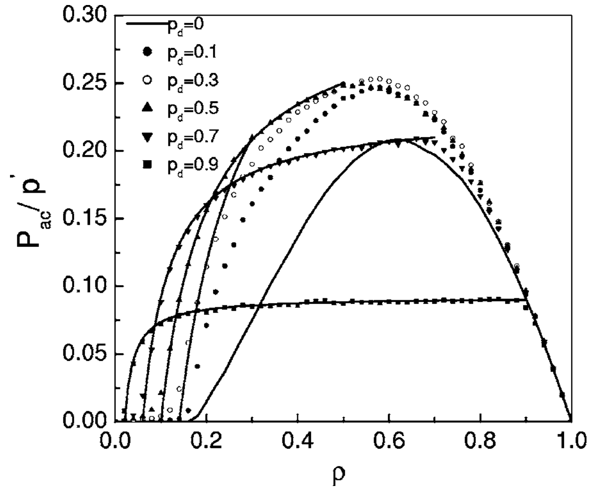

FIG. 1. Probability P_{ac} (scaled by p') as a function of the density ρ in the deterministic NS model with a defect for the case $p = 0$ and $v_{\max} = 5$. Solid lines correspond to analytical results, and symbol data are obtained from numerical simulations.

III. NUMERICAL RESULTS

A. Effects of the defect in the deterministic model

First, we investigate the influence of the defect on the probability P_{ac} in the deterministic case. In the deterministic NS model with the defect, the stochastic braking of the drivers is not considered, i.e., $p = 0$. Figure 1 shows the accident probability P_{ac} as a function of ρ for various values of p_d. In Fig. 1, there is a critical density below which no car accident happens, above which the probability for a car accident P_{ac} increases with the increase of ρ, and reaches a maximum, but decreases with further increase of ρ. The density for the onset of P_{ac} is usually named as the critical density ρ_{c_1}.

Apparently, effects of the probability of braking at the defect p_d on the accident probability P_{ac} are great. As shown in Fig. 1, the braking probability p_d influences not only the critical density ρ_{c_1} but the value of the probability P_{ac}. With the increase of the braking probability p_d, the critical density ρ_{c_1} decreases accordingly. The results are easily understood. According to the previous studies for traffic flow in the NS model with a defect [20,21], there is a critical density above which the system exhibits a saturated traffic flow, and the localized blockage has global effects whereby the traffic exhibits macroscopic phase segregation into high-density and low-density regions. The critical density ρ_{c_1} shifts toward the low-density region with the braking probability P_d increasing. As prescribed in the conditions of car accidents, the occurrence of car accidents is directly related to traffic flow and stopped cars [7,10], car accidents only happen in the high-density region, therefore the critical density ρ_{c_1} decreases with the increase of p_d. In the low-density region, there is no stopped car, therefore no car accident occurs.

The most remarkable results of our investigation show that when the braking probability p_d is very small, the defect induces many more accidents; while when the probability p_d is large, the defect suppresses the occurrence of car accidents, as shown in Fig. 1. The simulation results seem to be contrary to our intuition of ordinary life which the limited velocity of cars at one point will always suppress the occurrence of car accidents. In fact, since the probability for an accident is directly related to traffic flow and stopped cars, when the braking probability p_d is small, increasing p_d leads to a larger value of the fraction of stopped cars n_0, but small changes of traffic flow $\langle J\rangle$, thus the probability P_{ac} increases with the increase of the probability p_d. However, when the value of p_d is larger, increasing the braking probability p_d leads to a decrease of traffic flow, consequently a decrease of the occurrence of car accidents.

The simulation results above can be quantitatively explained. The earlier results indicate that the probability for the occurrence of car accidents P_{ac} is proportional to the product of traffic flow $\langle J\rangle$ and the fraction of the stopped cars n_0 [10]. When the density ρ is small, all cars freely move at $v_{\max}$ sites at every time step, no stopped car exists, hence no car accident occurs.

However, the density ρ further increases over the critical density ρ_{c_1}, traffic jams begin to emerge. In the density interval $\rho_{c_1}<\rho<\rho_{c_2}$, the traffic flow remains in the saturated value $\langle J\rangle_s$, because the bottleneck at the impurity is the flow-limiting factor. Therefore in this regime, $\langle J\rangle_s$ follows the relation

$$\langle J\rangle_s = 1 - p_d. \tag{1}$$

Apparently, at the critical density ρ_{c_1} the mean velocity $\langle v\rangle$ reads as

$$\langle v\rangle = v_{\max} = \frac{1-p_d}{\rho_{c_1}}. \tag{2}$$

Thus the critical density $\rho_{c_1}=(1-p_d)/v_{\max}$. Increase of the value of p_d causes a decrease of value of ρ_{c_1}. This analytical formula well illustrates the simulation results for the critical density shown in Fig. 1.

When the value of the density ρ is larger than the value of the critical value ρ_{c_2}, the vehicle queuing extends all of the road, hence the phase separation vanishes. The effect of the defect becomes negligible. Actually, in the case of the high-density region, the drivers move only one site at every time step. According to the definition about the braking probability p_d, the probability p_d means the probability for a car to be stopped in the case of high density. Thus $1/p_d$ indicates the time interval during which one car stays at the defect. Apparently, when $1/\rho$, which means the number of empty sites between two near cars, is larger than the number of empty cells between the defect and its left near car $(1/p_d)\times 1$, where 1 denotes the car's velocity, the traffic flow is limited by the defect; otherwise, the traffic flow is controlled by the density. Therefore the critical density $\rho_{c_2}=p_d$. Our theoretical analyses about the traffic flow and the critical density ρ_{c_1} and ρ_{c_2} are in good agreement with numerical results (these data are not shown). We also notice that the analyses about traffic flow in the NS model with a defect in the case of $v_{\max}>1$ have not been reported so far.

Now to derive explicit expression for the probability P_{ac}, we turn our attention to the fraction of the stopped cars. In the density interval $\rho_{c_1}<\rho<\rho_{c_2}$, conservation of the vehicles demands that

$$\rho L = \rho_{c_1}(L-L_q) + \rho_{c_2}L_q, \tag{3}$$

where L_q denotes the length of vehicle queuing, also the length of high density; $L-L_q$ denotes the length of low density. The formula (3) demonstrates that the number of cars in the region of both high density and low density is the total number of cars. Only in the high-density region, stopped cars appear. The density of stopped cars in the high-density regime can be obtained by considering that the probability for a site to be occupied by a stopped car p_d is divided by the empty space between the stopped car and another one $(1/p_d)\times 1$, therefore the density of the stopped cars follows the relation

$$\frac{N_0}{L_q} = p_d \bigg/ \left(\frac{1}{p_d}\times 1\right) = p_d^2. \tag{4}$$

Substituting formula (4) into Eq. (3), and using the relation $n_0=N_0/N$, we obtain the expression of the fraction of stopped cars:

$$n_0 = \frac{(\rho-\rho_{c_1})p_d^2}{(\rho_{c_2}-\rho_{c_1})\rho}. \tag{5}$$

According to the previous studies, the probability for the occurrence of car accidents is proportional to the product of the traffic flow and the fraction of the stopped cars, therefore the probability for car accidents P_{ac} is given by

$$P_{ac}/p' = p_d^2\frac{(1-p_d)(\rho-\rho_{c_1})}{(\rho_{c_2}-\rho_{c_1})\rho}. \tag{6}$$

Good agreement between the numerical results and our theoretical analyses for the value of P_{ac} is obtained, especially in the case of the high braking probability. These results are shown in Fig. 1. According to formula (6), when the density ρ is smaller than the critical density ρ_{c_1}, there is no car accident. However, when the value of ρ is larger than the value of ρ_{c_1}, car accidents happen, and the probability P_{ac} varies nonlinearly with the increase of the density ρ. Moreover, formula (6) demands that $\rho_{c_2}-\rho_{c_1}>0$ because of no minus value of the probability for car accidents. Thus Eq. (6) is only suitable for the situation of the braking probability

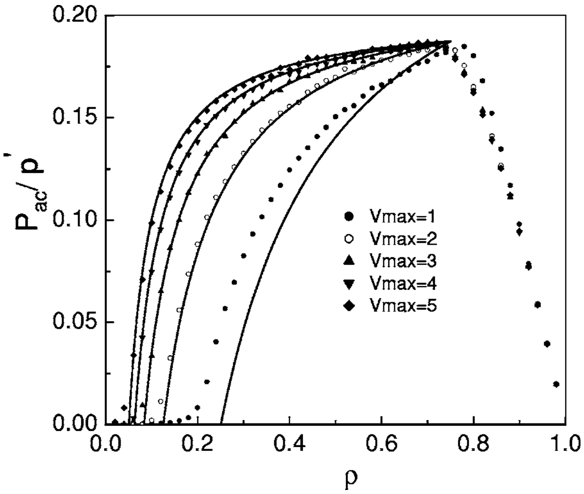

FIG. 2. Probability P_{ac} (scaled by p') as a function of the density ρ in the deterministic NS model with a defect for various values of the maximum speed $v_{\max}$. Solid lines are analytic results, and symbol data are obtained from numerical simulations.

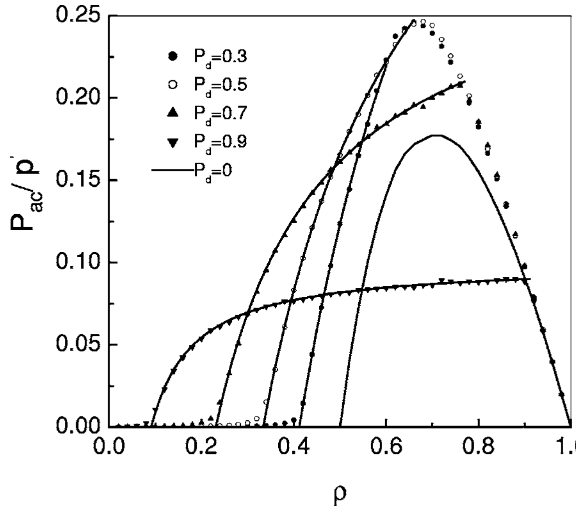

FIG. 3. The relation of probability P_{ac} (scaled by p') to the density ρ in the deterministic NS model with $v_{\max}=1$. Symbol data are obtained from computer simulations, and the solid line corresponds to analytic results.

$p_d > 1/(1+v_{\max})$. Therefore in the case of $p_d=0.1$ for $v_{\max}=5$, the theoretical results do not coincide with the computer simulations.

But in the case of $\rho > \rho_{c_2}$ and low braking probability at the defect, the effects of the defect can be ignored, the curves of the probability P_{ac} do not collapse in the curve of the probability for the NS model without a defect unless $\rho \to 1$. This number result indicates that the system exhibits different correlations from the one without a defect in the process of car accidents, although the relationship between traffic flow and bulk density is the same as that in the model without a defect. When the density is near 1, the accident probability P_{ac} is proportional to $\rho(1-\rho)$.

Obviously, the probability for accidents P_{ac} is not only related to the braking probability p_d, but also determined by the velocity limit $v_{\max}$ when the stochastic braking probability $p=0$. Figure 2 exhibits the relation of P_{ac} to ρ with different values of the speed limit $v_{\max}$. In Fig. 2, the critical density ρ_{c_1} which denotes the onset of car accidents, decreases with the increase of the speed limit. We have also observed in Fig. 2 that the probability P_{ac} is also increased with the increase of the speed limit. The phenomena can be well explained for our theoretical results. According to the results presented above, the critical density $\rho_{c_1}=(1-p_d)/v_{\max}$ reveals that increasing the value of $v_{\max}$ leads to the decrease of the value of ρ_{c_1}. As the value of the speed limit $v_{\max}$ increases, the number of the stopped cars increases, and causes more car accidents to happen. As expected, the formula (6) gives exact results for the value of P_{ac} except in the case of $v_{\max}=1$, seen in Fig. 2.

For $v_{\max}=1$, the particle-hole symmetry in the special case exists. For $\rho < \rho_{c_1}$, the flux is $\langle J\rangle=\rho$, and for $\rho > \rho_{c_2}$, the flux is $\langle J\rangle = 1-\rho$. However, for $\rho_{c_1} < \rho < \rho_{c_2}$, the flux remains constant. The critical density ρ_{c_1} and ρ_{c_2} are different from those for the case of $v_{\max} \neq 1$. In Ref. [24], the value of the critical density ρ_{c_1} and ρ_{c_2} are given, respectively, by

$$\rho_{c_1} = \frac{1-p_d}{2-p_d}, \quad \rho_{c_2} = \frac{1}{2-p_d} \tag{7}$$

and the traffic flow reads as

$$\langle J\rangle = \frac{1-p_d}{2-p_d}. \tag{8}$$

In the density interval $\rho_{c_1} < \rho < \rho_{c_2}$, the system is separated into two regions of constant density. However, stopped cars only appear in the high-density region. Owing to the flow limited by the capacity of the defect site and the symmetry of the particle hole, the density of the stopped cars is the value of the braking probability p_d, i.e.,

$$\frac{N_0}{L} = p_d. \tag{9}$$

Substituting formula (9) into Eq. (3), we derive the relation of the fraction of the stopped cars n_0 to the density of the cars ρ:

$$n_0 = \frac{(\rho-\rho_{c_1})p_d}{(\rho_{c_2}-\rho_{c_1})\rho}. \tag{10}$$

Therefore the probability for the occurrence of car accidents P_{ac} follows the relation below:

$$P_{ac}/p' = p_d \frac{(1-p_d)(\rho-\rho_{c_1})}{(2-p_d)(\rho_{c_2}-\rho_{c_1})\rho}. \tag{11}$$

Figure 3 shows the relation between the probability P_{ac} and the density ρ with various values of p_d. As shown in Fig. 3, formula (11) gives good agreement with the simulation data.

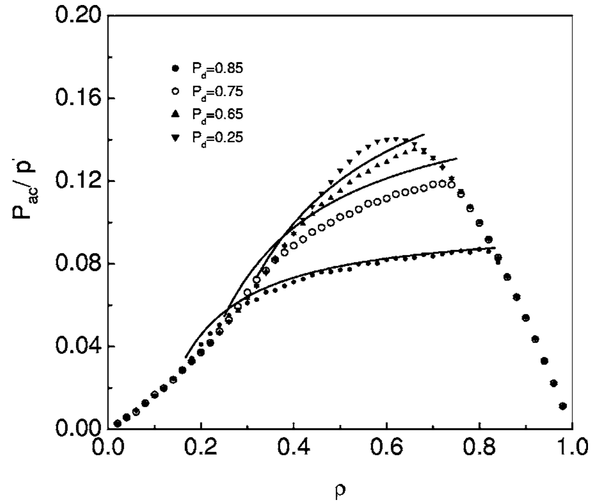

FIG. 4. Probability P_{ac} (scaled by p') as a function of the density ρ in the nondeterministic NS model with $v_{\max}=1$. Solid lines are obtained from explicit expression (16), and symbol data are numerical results.

B. Effects of the defect in the nondeterministic case

Next, we investigate the probability for the occurrence of car accidents when the stochastic braking behaviors of drivers are considered, i.e., $p \neq 0$. Figure 4 shows the relations of the probability P_{ac} to the density ρ with various values of the braking probability p_d in the case of $v_{\max}=1$, where the randomization parameter p_d at the blockage is larger than that of the rest of the system. Compared to the results in the case of $v_{\max}>1$, the probability displays different behaviors. In the system with $v_{\max}=1$, some cars can be stopped due to the stochastic braking, even the car density is very small, therefore car accidents happen in the whole density region, while the critical density exists in the systems with $v_{\max}>1$. Moreover, the effects of blockage probability p_d on the accident probability P_{ac} are much different from those in the deterministic NS model As shown in Fig. 4, the probability P_{ac} is suppressed by increasing the values of the probability p_d especially in the density region $\rho_{c_1}<\rho<\rho_{c_2}$, where the traffic flow is maximum and independent of the car density ρ and the traffic displays phase separation of high density from low density. In this density region, the traffic flow is limited by the blockage, as prescribed by the three conditions for the occurrence of car accidents, the occurrence of car accidents is directly related to traffic flow and stopped cars, therefore the probability P_{ac} is diminished. Outside the density region, because the bottleneck does not influence the traffic flow, the probability P_{ac} exhibits the same behaviors with that in the system without bottlenecks.

The critical density ρ_{c_1} and ρ_{c_2} can be given by [1]

$$\rho_{c_1}=\frac{q_d}{q+q_d}, \quad \rho_{c_2}=\frac{q}{q+q_d} \tag{12}$$

and the traffic flow

$$J=\frac{1}{2}\left(1-\sqrt{1-\frac{4q^2q_d}{(q+q_d)^2}}\right). \tag{13}$$

Therefore in the high-density region, the fraction of the stopped cars can be given

$$n_{0h}=1-\frac{J}{\rho_{c_2}} \tag{14}$$

and the fraction of the stopped cars in the low-density region reads

$$n_{0l}=1-\frac{J}{\rho_{c_1}}. \tag{15}$$

In the density region $\rho_{c_1}<\rho<\rho_{c_2}$, the probability for the occurrence of car accidents per car per time step in the high-density and low-density region can be read as Jn_{0h} and Jn_{0l}, respectively. Therefore the probability P_{ac}/p' can be given by

$$\begin{aligned} P_{ac}/p' &= J(n_{0h}N_h+n_{0l}N_l)/N = J(n_{0h}\rho_{c_2}h+n_{0l}\rho_{c_1}l)/N \\ &= J(n_{0h}\rho_{c_2}h/L+n_{0l}\rho_{c_1}l/L)/\rho, \end{aligned} \tag{16}$$

where N_h and N_l denote the car numbers in the high-density region and low-density region, respectively, and h and l mean the lengths of high-density region and low-density region. According to the previous results, h/L and l/L are respectively given by [24]

$$h/L=\frac{\rho\left(1+\frac{q_d}{q}\right)-\frac{q_d}{q}}{1-\frac{q_d}{q}},$$

$$l/L=1-h/L. \tag{17}$$

Figure 4 gives a comparison between number results and Eq. (16). As shown in Fig. 4, the agreement can be obtained in the case of large value of the defect braking probability. But in the case of small defect probability of braking, Eq. (16) overestimates the accident probability P_{ac}. The differences may be from the transition region from the high to the low density.

Figure 5 shows the relations of probability P_{ac} to the density ρ with various values of the braking probability p_d, where the randomization parameter p_d is larger than that in the rest of the system. As shown in Fig. 3, the effects of the defect on the probability for accidents P_{ac} show similar behavior with those in the deterministic case. As the defect probability p_d increases, the probability P_{ac} increases accordingly, and decreases with further increase of p_d. And the critical density ρ_{c_1} shifts toward the low-density region with the increase of the braking probability p_d.

Quantitative expression for the probability P_{ac} is difficult to be obtained, because the stochastically delayed probability induces the long length correlations of time-space correlations.

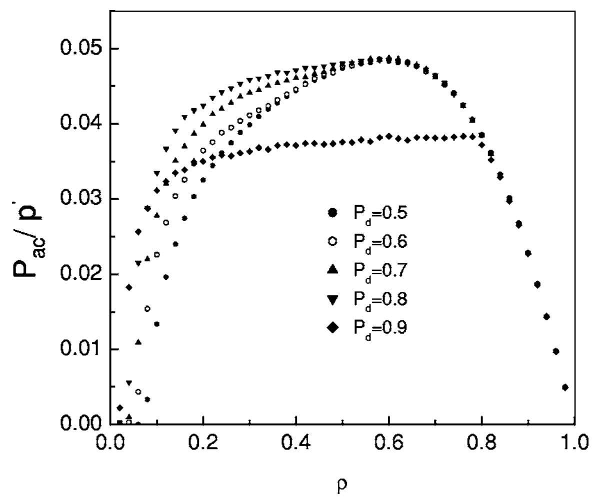

FIG. 5. Probability P_{ac} (scaled by p') as a function of the density ρ in the nondeterministic NS model with $v_{\max} \neq 1$.

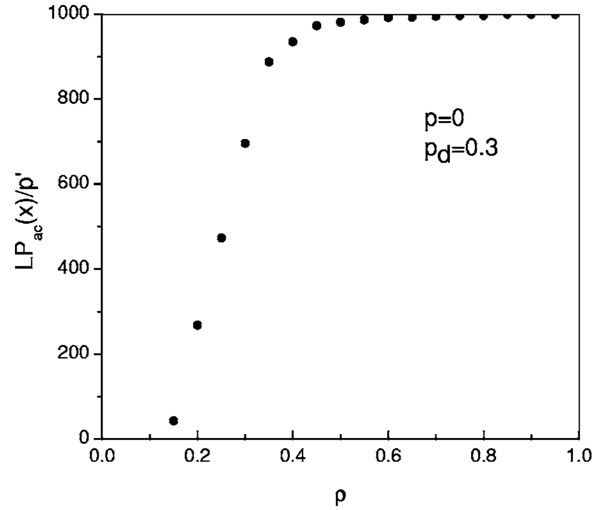

FIG. 7. Accident length LP_{ac} (scaled by p') as a function of the density ρ in the case of $v_{\max}=5$.

C. Position distribution of P_{ac} induced by the defect

As the braking at the bottleneck results in the inhomogeneous density distribution of cars, unlike that of a model without defects, the occurrence of car accidents depends on the sites. Figure 6 shows the relations of the accident probability to sites, the parameters correspond to the maximum traffic flow and phase segregation into high density and low density. As shown in Fig. 6, car accidents happen upstream of the bottleneck, and the probability SP_{ac}/p' exhibits very small fluctuation around a fixed value. The dependence of the accident probability P_{ac} on sites can be easily understood. The high density appears upstream of the bottleneck, hence car accidents happen; while downstream of the bottleneck, there is low-density region, no car accident takes place, because of free flow and no stopped cars.

As the car density increases, the length of region where car accidents happen expands because length of the high-density region increases. As shown in Fig. 7, the length LP_{ac} of accident region displays a linear increase with the density until the critical density ρ_{c_2}. When $\rho > \rho_{c_2}$, the occurrence of car accidents extends to the whole road.

But, the value of the probability SP_{ac}/p' at high-density sites approaches a constant with the density increasing until $\rho > \rho_{c_2}$. Figure 8 exhibits the accident probability SP_{ac}/p' at the high-density sites varies with the density ρ. In Fig. 8, the probability in the density interval $\rho_{c_1} < \rho < \rho_{c_2}$ shows to be an approximate constant, because traffic flow and stopped cars in this region do not vary. When $\rho > \rho_{c_2}$, the probability increases with the increase of the car density, reaches to a maximum, but decreases with further increase in density.

IV. SUMMARY

In this paper, we study the probability for the occurrence of car accidents in the NS model with a defect. According to the previous studies of car accidents, the probability of car accidents is directly related to stopped cars and traffic flow

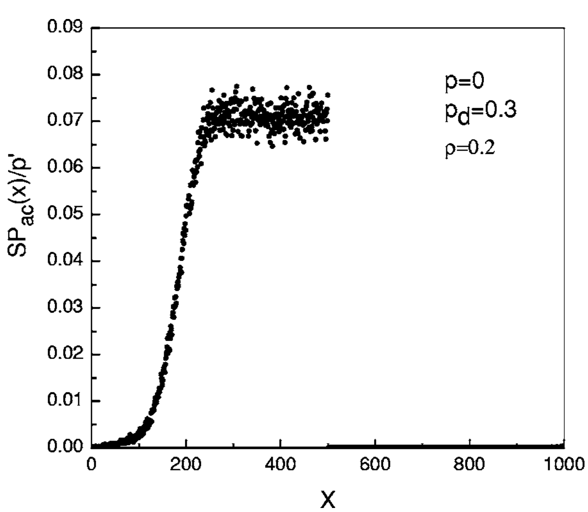

FIG. 6. Distribution of the accident probability P_{ac} (scaled by p') in the deterministic NS model with $v_{\max}=5$.

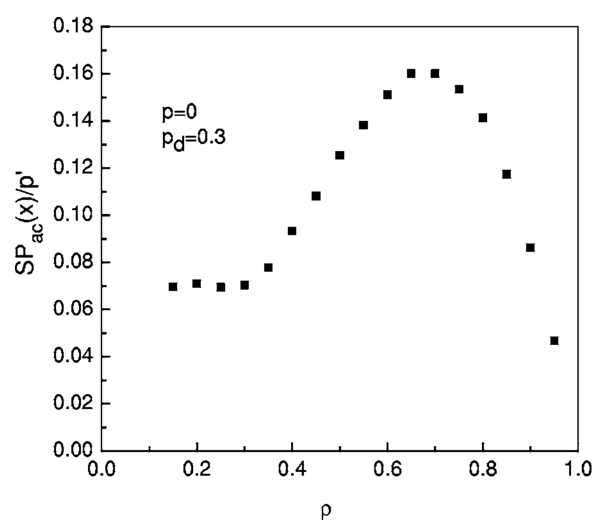

FIG. 8. Probability SP_{ac} (scaled by p') at the sites as a function of the density ρ in the case of $v_{\max}=5$.

[10], the studies on the probability of the occurrence of car accidents can lead us to understand traffic flow. On the other hand, because the bottleneck results in a distinct variety of traffic flow such as a saturated flow observed in a plateau region in the fundamental diagram, and vehicle queuing, changes of traffic flow and number of stopped cars in a model with a bottleneck certainly will influence the probability of the occurrence of car accidents. Therefore car accidents in a CA model with a bottleneck should be further investigated.

Numerical results show that there is a critical density below which no car accident happens. The critical density ρ_{c_1} is not related only to the maximum speed of cars, but also to the braking probability at the defect. The defect can enhance, not suppress, the occurrence of car accidents when its value is small. Only the braking probability at the defect is very large, car accidents can be reduced by the defect. But in the case of $\rho > \rho_{c_2}$ and low defect braking probability, the number results exhibit the system exhibits different correlations from the one without a defect in the process of car accidents, although the relationship between traffic flow and bulk density is the same as that in the model without a defect.

The probability for accidents P_{ac} is not only related to the braking probability p_d, but is also determined by the velocity limit $v_{\max}$ when the stochastic braking probability $p=0$. The probability P_{ac} is enhanced with the increase of the speed limit, because increasing the value of $v_{\max}$ induces the increase in the number of the stopped cars, and hence the increase in car accidents.

In the nondeterministic NS model, the probability P_{ac} exhibits the same behaviors with that in the deterministic model except the case of $v_{\max}=1$ under which the probability P_{ac} is only reduced by the defect. As the defect probability p_d increases, the probability P_{ac} increases accordingly, and decreases with further increase of p_d. And the critical density ρ_{c_1} shifts toward the low-density region with the increase of the braking probability p_d.

The defect also results in the inhomogeneous distribution of car accidents over the whole road. Car accidents happen only in the high-density region. In the region of maximum traffic flow, as the density increases, the length of the occurrence of car accidents displays a linear increase, but the value of the accident probability shows to be an approximate constant until the critical density ρ_{c_2}.

A phenomenological mean-field theory is presented to describe the accident probability P_{ac} in deterministic NS model. Theoretical analyses give an excellent agreement with numerical results in a deterministic NS model with a defect. In a nondeterministic NS model with $v_{max}=1$, we also obtained the explicit expression only suitable for the case of large defect braking probability. But in the nondeterministic NS model with $v_{\max}>1$, explicit expressions which deserve further investigation are not obtained because of effects of long length of time-space correlations.

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